

# ***Literature Review – NewSchools Ignite Middle & High School Math Challenge***

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## Overview

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Educational technology developers have an unprecedented opportunity to promote learning and address specific needs of teachers and students in mathematics. Addressing these needs using technology requires a combined understanding of both the best practices for instruction and learning in mathematics education, and how technology can best be used to promote those best practices for students and teachers in mathematics.

Research findings in cognition and mathematics education can greatly inform the efforts of edtech developers. From these findings, developers have a better understanding of evidence-based best practices in mathematics education and how they can be promoted through the use of technology. From using representations, visualizations, and graphics to quizzing and supporting sense-making and discourse, research findings provide a roadmap for technology developers to follow in addressing the needs of mathematics students and teachers. Key features of effective mathematics edtech that support activities that research suggests are highly likely to promote student learning include: formative assessment, productive mathematical discourse, connecting abstract and concrete representations, active participation, integrating visual and verbal information, and improved pedagogical content knowledge.

Education technology can support these activities through networked technology and teacher professional development. Education technology enables formative assessment in the form of activities, polls and quizzes; simulation software supports integration between multiple representations, and allows the anonymous sharing of student and group responses. In professional development, teachers should build their content understanding working with the technology as a learner, then shift and build their pedagogical content knowledge (PCK) by examining common classroom situations, analyzing student work, and planning for their own instruction. Building PCK can thus be extended to technology pedagogical content knowledge (TPACK).

## Need

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As the world becomes more technologically advanced, the need for computational reasoning and problem solving intensifies. Mathematics proficiency is one of the major gateways for access to college, citizenship and economic access, and educators need access to instructional tools that will improve mathematical understanding. According to Achieve, Inc., there is an *expectation gap*—a sizeable gap between what students knew leaving high school and the actual knowledge and skills they need to be successful in college and careers. In 2015, thirty-three percent of eighth-grade students performed at or above the *Proficient* level on the National Assessment of Educational Progress (NAEP) mathematics assessment, which was 2 percentage points lower compared to 2013, the previous assessment year. Twenty-five percent of twelfth-grade students performed at or above the *Proficient* achievement level in mathematics.

Algebra, in particular, serves as a gatekeeper to more advanced mathematics and performance in algebra has been linked to success in college and long-term earnings potential. Students failing to achieve math proficiency in middle school typically repeat classes in high school, with progressively poorer outcomes. Longitudinal studies also indicate that students taking rigorous

high school mathematics courses are twice as likely to graduate from college than those who do not.

Mathematics achievement in the U.S. is not keeping pace with international performance either. Comparative data from studies such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) show that U.S. students perform at or below average relative to students of many industrialized nations. Similarly, NAEP results show that: (1) U.S. student achievement continues to fall below desired proficiency levels in grades 4, 8, and 12, (2) achievement gaps persist among gender and racial groups, and (3) although we have begun to see improvement in the mathematics achievement levels of students in grade 4, there is correspondingly little evidence of similar achievement gains over time for students in grades 8 or 12.

## Best Practices in Mathematics Education:

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A number of research-based best practices for teaching mathematics have been identified and described in the research literature. Among those most applicable to the design and development of education technology are:

*Visualizing and Representing.* Representations can be powerful aids to student learning. Developing mathematical meaning involves recognizing an idea in a variety of representations and translating among them. Generating and discussing visual representations of abstract mathematical ideas supports flexible thinking and connections among mathematical ideas. Laboratory studies in cognitive science provide evidence that connecting abstract and concrete representations of problems improve learning. In mathematics, research has demonstrated that interleaving graphics with visual descriptions promotes deeper understanding.

*Combine Graphics with Verbal Descriptions.* Combining graphics with verbal descriptions serves two important functions in mathematics instruction: 1) ensuring that text for instruction and problem-solving is perceived and understood and 2) promoting fluency in mapping between representations. The multimedia learning literature has demonstrated that adding relevant diagrams and pictures to text-based materials leads to better learning than text alone. To maximize the learning benefits, research suggests that text labels should be in close proximity to the relevant parts of pictorial representations. Further, the diagram or picture should be directly relevant to the content of instruction and avoid “seductive details;” that is, representations that are engaging but only tangentially related to the topic of instruction or the problem at hand.

A second function of combining graphics with verbal descriptions is to promote fluency in mapping between representations. Analyses of the prerequisites for success in mathematics strongly suggest that strategic competencies, such as the ability to translate among verbal, symbolic, graphic, and pictorial representations, may be essential not only for algebra but for other areas of mathematics as well. As noted by Kilpatrick et al., “Understanding a mathematical idea thoroughly requires that several possible representations be available to allow a choice of those most useful for solving a particular problem”. Different representations of the same concept provide access to different information and possibilities about mathematical ideas. Thus, the ability to understand and use models, diagrams, symbolic expressions, and language plays an integral role in students’ understanding of algebra as well as arithmetic, particularly in the context of recent reform efforts.

Combining graphics with verbal descriptions can be applied to various aspects of a curriculum including learning materials, practice exercises, and classroom instruction. For example, studies on math learning show that students learn more when activities require students to use graphical representations of core concepts. Multiple classroom studies have demonstrated that the use of a number line visualization leads to increased learning of core numerical concepts. Graphics and verbal descriptions can be effectively combined in instructional materials by highlighting relationships using proximity (e.g., verbal labels on aspects of diagrams or graphs), color (e.g., highlighting corresponding aspects of related representations in the same color), or other forms of connections (e.g., arrows to connect corresponding aspects of representations). Another avenue to combine graphics with verbal descriptions is during classroom instruction. With appropriate guidance, teachers can increase their use of gestures that simultaneously link corresponding aspects of multiple representations. These studies suggest that combining graphics with verbal descriptions by carefully directing student attention to the appropriate correspondences and providing guidance to teachers on appropriate instructional moves are promising approaches to guide the design of mathematics curriculum and instructional resources. This principle can be applied to text materials, practice activities, and recommendations for teachers.

*Structure Practice to Interleave Worked Examples.* Cognitive theories of skill acquisition place great importance on practice because it leads to fluency and a reduction in the amount of processing resources needed to retrieve knowledge and execute a cognitive skill. In mathematics, students must learn to fluently carry out procedures across a variety of problem types. Effective co-development of conceptual *and* procedural knowledge may be improved by interleaving worked examples into problem-solving activities and prompting students to self-explain responses. In worked example exercises, students study solutions rather than solve problems themselves. Positive effects of interleaving worked examples have been reported in a variety of courses. Worked examples are more effective *and* more efficient for learning and transfer because they allow students to spend limited cognitive resources on understanding the ideas underlying the solutions rather than on generating solutions. Prompting students to “self-explain” while studying examples or texts improves learning and transfer by helping students to integrate pieces of knowledge. In a typical worked example exercise, students see a correct solution and explain why it is correct. However, explaining incorrect problem solutions can also improve learning outcomes. Asking students to explain why incorrect answers are incorrect is common in Japan, where mathematics achievement is outstanding by world standards. Further, a number of empirical laboratory studies confirm that asking students to explain incorrect, as well as correct, solutions leads to greater learning. Interleaving both correct and incorrect types of worked examples with problems challenges students’ illusions of knowing, forces active knowledge construction and retrieval, and reveals misconceptions that can be remediated through teacher or tutor feedback.

*Space Learning over Time.* Extensive research in cognitive psychology has demonstrated large retention advantages when learners are exposed to key facts, concepts, and knowledge at multiple points in time, a phenomenon called the spacing effect. When learners practice recalling and applying relevant information, they are more likely to retain that knowledge for a greater period of time. The benefits of distributed practice have been known for quite some time, based largely on laboratory studies.

In classroom learning, spacing instruction and practice reinforces connections between key ideas and promotes transfer. In a review of the literature on the spacing effect, Rohrer argues that practice is neglected in mathematics education, as other authors have concluded. Students learn more when questions are distributed over time compared to when questions are all given at the same time, even though the total number of questions remains fixed. While students in a spacing condition performed less well during training than students who received all the training examples massed together, their post-test performance was superior. Kornell and Bjork call these situations “desirable difficulties” that may decrease students’ performance during acquisition but lead to longer term retention benefits. Another desirable difficulty is mixing problem types instead of massed practice on items of the same type; Rohrer found the best results have been obtained by using spacing and mixing together.

*Use Quizzing to Promote Learning.* Research suggests that these activities are successful in part because they prompt students to recall information, reflect on the state of their knowledge and understanding, and offer opportunities to transfer knowledge to new problems or situations. Feedback, revision, and reflection are also important aspects of meta-cognition and as such are critical to developing the ability to regulate personal learning.

Periodic testing provides students with opportunities to practice retrieving knowledge and using skills and concepts. It has long been known that practice alone without knowledge of results is much less effective than practice with feedback on how to adjust performance. However, learners may lack the knowledge to generate their own feedback and may waste time practicing incorrect skills. Well-designed cycles of feedback and reflection with opportunities for revision and knowledge updating can support students in practice that leads to mastery of the desired skills and concepts. Much of the work cited in support of the beneficial effects of quizzing and testing comes from laboratory studies focused on content that is largely declarative in nature with relatively few studies of the types of content and skills represented by school subject matter. There is a substantial body of evidence from classroom learning contexts showing that the *formative* use of assessment can enhance instructional effectiveness; here, formative assessment is defined as a process used by teachers and students that provides feedback to adjust ongoing teaching and learning to improve students’ achievement of intended instructional outcomes. Nyquist examined the role of feedback in formative assessment in a meta-analysis of over 100 studies that manipulated levels of feedback on tests administered within instructional settings. Across studies, the more detailed the feedback, the more students profited from having been tested. Directive and clear feedback, which included correct results combined with activities to correct errors, resulted in over half a standard deviation of improvement in student learning. These findings are consistent with the synthesis of research on formative assessment reported in Black & Wiliam. Wiliam suggests that using assessment formatively may be the most consequential component in the effectiveness of any instructional program. Educators are only beginning to understand the power of formative assessment, most especially the conditions of use to strategically improve instructional practice and student learning.

Given the empirical data on testing effects and formative assessment, effective implementation of mathematics curricula depends on the presence of embedded, high quality assessments of student learning tied to cognitive models of student knowledge and understanding. Virtually every curriculum, and especially those developed to align with NCTM standards (1989, 1995, 2000), includes a wide range of assessment materials with potential uses that span the range from

formative assessment within lessons to summative assessment across sets of lessons or major units.

*Confronting Pitfalls.* Presenting students with a math problem accompanied by a correct solution and a solution containing a common student error is an effective way to introduce concepts to novice learners. Contrasting the reasoning underlying correct solutions and solutions with pitfalls (common flaws) builds conceptual meaning and prompts students to self-monitor, self-assess, and self-correct. Specifically, contrasting correct and incorrect solutions in mathematics promotes conceptual understanding, retention, and self-correction. Examples and non-examples are useful in forming concepts while metacognition, self-monitoring or awareness of one's own thinking, is associated with increased learning. The inability to solve problems with misleading features is symptomatic of fundamental misunderstandings. Confronting pitfalls is a pedagogical technique regularly used in several high-achieving countries, but US teachers generally do not use students' errors productively during mathematics lessons.

*Sense-Making.* Making sense of problems that are in context, as well as those expressed only in symbols, is central to understanding, enjoying, and using mathematics. Research on self-explanation suggests that generating explanations while problem solving promotes conceptual understanding as the process encourages students to generate inferences to fill in missing information, integrate information with prior knowledge, and monitor and repair faulty knowledge. In mathematics, the research of Carpenter, Fennema, Peterson, Chiang, & Loef verifies the role of word problems in providing meaningful context for developing mathematical understanding. Sfard advises that meaningful and skillful work with symbolic mathematics is essential for higher mathematics.

*Building Discussions.* Engaging in rigorous, respectful discussion builds mathematical understanding. Knowledge is socially constructed and participation in a discourse community is key in developing meaning. Equitable opportunities to participate in discourse and learning depend on attending to issues of culture, language, and status.

*Capturing Key Ideas.* Capturing a public record strategically during a class discussion focuses attention on key mathematical ideas and helps students understand, link related concepts, summarize, and remember. A public record focusing attention on key mathematical ideas provides a mediating tool for collaborative knowledge building and shared meaning. The role of scaffolding may be central to supporting new cognitive behaviors and patterns of discourse. Effective instruction must be guided by a vision for where tasks might lead in terms of a "hypothetical learning trajectory" and a public record of developing understandings may assist teachers to monitor and adjust this trajectory.

*Professional Development and Support.* Professional development on the use of mathematics instructional materials should draw on principles of effective professional development. These works suggest that professional development must be consistent with local curriculum and standards and focus on strengthening teachers' content and pedagogical content knowledge. Also, there is mounting evidence that the duration of the professional development and the building of a community of learners contributes to teacher learning, change in practice, and increases in student learning. "Teachers as learners need time to activate their prior knowledge, instruction that builds deep understanding of key ideas and to make connections to supporting concepts and facts, and they need to be metacognitive about their learning". Professional

development begins by activating what teachers already know about the content and strategies they will learn.

## Applying Educational Technology to Mathematics

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Learning and instruction in mathematics has benefitted from technology advances in other fields, which can then be applied to mathematics. Most often these technologies target practices that are common to both mathematics and science and engineering. For example, both the Standards for Mathematical Practice in Common Core Math and the Science and Engineering Practices in the Next Generation Science Standards (NGSS) call for developing and using models. One way for supporting this is by using simulations. Another practice common to both Common Core and NGSS focuses on argumentation. Technologies that allow for collaboration and interactions with other students promote this practice in both mathematics and science.

*Access to Learning.* Technologies have caused a paradigm shift in education away from a one-way flow of information (the teacher as the sage on the stage) to a collaborative interactive traffic of information and teaching between students and the teacher. These technologies include a host of Web 2.0 online tools that foster communication, collaboration, social and learning networks, as well as accessing information. They also include interactive whiteboards, tablet PCs, projectors and other tools that allow schools to present information in ways that encourage discussion and collaboration. Through the use of virtualization, online gaming, podcasting, virtual labs and more, the education technology provides an experience that is no longer limited to a physical space. As the usage of handheld computational tools with networking capabilities such as smartphones and other mobile devices continues to grow, educators need more rigorous information about the impact of education technologies on content learning to inform decisions about how to integrate these interactive technologies.

The digital divide is closing. Connectivity and access will soon be a non-issue in K-12 students' ability to engage in digital learning. President Obama's ConnectED 2013 initiative, tasks the Federal Communications Commission (FCC) with connecting 99% of America's K-12 students to gigabit broadband and robust Wi-Fi by 2018. This will transform the classroom experience for all students, regardless of income. "Between 2015 and 2020, hardware, software, and network technologies will mature sufficiently such that educational technology's Holy Grail for K-12--a computing device--a mobile device--for every child, 24/7--will be realized". Stevenson et al., propose that smart mobile devices provide "situated, authentic and connected" learning experiences and that apps should be explored as "cognitive stepping stones".

*Connecting Students and Teachers.* Education technologies allow for shared learning experiences by connecting teachers with their students. Many of these technologies allow a teacher to see all students' responses simultaneously, enabling them to provide rich interactive and differentiated formative assessment and evaluate student progress in real time throughout an activity. Teachers can monitor, display, and review, individually or collectively, the students' calculator screens, distribute and collect documents, create and monitor content, administer polls and quizzes, and grade and record students' work. Real-time polling can be utilized by instructors to implement teaching methodologies into their classroom that will enhance their students' learning transfer.

Formative use of assessment, in which the results of the assessment are used to modify instruction, can enhance instructional effectiveness. Educators are beginning to understand the power of formative assessment, especially the conditions of use to strategically improve instructional practice and student learning. Without prompt feedback, learners may waste time practicing incorrect skills. Teachers who use education technologies receive student responses immediately and can adjust their instruction in the moment. Using interactive technologies empowers the teacher to leverage students' prior knowledge, assess conceptual understanding, and attend to student learning through questions and answers with immediate feedback.

*Simulations Connecting Abstract and Concrete Representations.* Education technology allows teachers and students to directly interact with computer simulations of equations and graphs and develop algebraic reasoning. Abstract relationships can be concretely depicted through animations of processes. For example, *slope* and *intercept* are key ideas in learning about linear equations, yet when introduced in symbolic equations they can be presented as abstract parameters that appear as a constant term and a coefficient. By explicitly integrating these parameters into concrete depictions of lines plotted in a Cartesian coordinate system, or entries in a two-column "T-table," students can see and manipulate parametric values with concrete results. This allows for opportunities to discuss the connections among the different types of representations that further supports the development of algebraic reasoning by making connections with concrete and abstract examples. Learning is enhanced when learners connect and interleave abstract and concrete representations. The dynamic visual display can also more readily support perceptual-motor grounding through dynamic demonstrations and hands-on, student-directed manipulation. Research suggests that learning is enhanced when complex concepts are initially grounded in a perceptual-motor experience.

## Developing Educational Technology for Mathematics

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In some cases, technology development can address areas in mathematics that have previously been problem areas. Using some of the principles described earlier, technology developers have been able to target areas of need that are specific to mathematics learning and instruction.

Broadly, education technologies support connections between learners and resources, between learners, and between learners and teachers. For today's active learners, the "rapid proliferation of smartphones and other mobile devices with networking capabilities suggests an increasing convergence between handheld computational tools and classroom connectivity that could significantly change the nature of mathematics instruction in the near future". Many companies are developing free or low-cost mobile applications that can both promote deep learning in mathematics (as well as other academic areas) and provide the similar types of interactions currently afforded by education technology. Using technology tools for mathematics "can assist students' problem solving," "provide dynamically linked representations of ideas" and "encourage general metacognitive abilities".

Compact, linked, digital devices are changing the ways teachers interact with students in the classroom. Networked classroom communication systems support students to be active participants in the learning process. Giving students an opportunity to interact and manipulate mathematical objects, "...help to lessen the distance between the student experience and the abstract mathematical concepts". This kind of technology facilitates a learner-centered classroom

where students receive timely feedback from their teacher, their peers, and the connected calculator. Such feedback can increase engagement and lead to a deeper understanding of complex subject matter. As student work is immediately aggregated, teachers get data about actual, urgent, and changing needs and are able to adjust their teaching. Pape et al. argue that connected classroom technology can be used as an important tool for creating contexts in which students engage in deep mathematical thinking and provides a mechanism for classroom communication that increases students' ability to engage with mathematics content. Irving et al. reported, "the implementation of random assignment with a true control group provides strong evidence to support the inference that the use of connected classroom technology by treatment teachers caused the increased algebra achievement".

*In-the-moment Formative Assessment.* The use of technology-enhanced formative assessment (TEFA) allows teachers to weave assessments into the learning environment by posing questions and receiving responses wirelessly from all students simultaneously. Research indicates that TEFA can be highly effective, even transformative, for mathematics instruction. "Technology that facilitates students' and teachers' immediate feedback in a public, non-threatening forum may improve the difficult task of formative assessment in the classroom and may support productive mathematical discourse in a classroom community". As students respond to questions using their connected calculator, the teacher is able to instantly see who has understood the content and who has not. This technology allows the teacher to use quizzing to re-teach key content. Research suggests that quizzing, questioning, and assessment activities enhance student learning because they prompt students to recall information, reflect on the state of their knowledge and understanding, and offer opportunities to transfer knowledge to new problems or situations. Self-regulated learning principles of feedback, revision, and reflection are also critical to developing the ability to monitor and improve personal learning. Beatty posits that connected classroom communication systems enable students to be active participants in the learning process by integrating new knowledge and overcoming misconceptions.

*Facilitating Productive Mathematical Discourse and Encouraging Active Participation.* Education technology makes possible new forms of participation—the amplified degree to which students are paying attention and their new, more substantial role in the classroom. In the networked environment, learning is promoted through mutual collaboration. Teachers encourage students to formulate and test their ideas with other students and to frequently assess how an activity is helping them gain math understanding. Research studies using audience response systems (ARS) in student-centered learning environments show conceptual gains.

*Integration of Verbal with Visual Information.* One affordance of education technology is the use of highly graphic and interactive modes to promote more frequent integration of visual and verbal information during instruction. Multiple representations can enhance learning, particularly when students are actively engaged in processing and linking the representations. Different representations of the same concept provide access to different information and possibilities about mathematical ideas. Thus, the ability to understand and fluently translate between graphs, diagrams, symbolic expressions, and language plays an integral role in students' understanding of algebra. In particular, combining graphics with verbal descriptions increases learning, presumably because encoding of information is enhanced when information is processed simultaneously through visual and auditory sensory channels. Further, dynamic displays have been shown to increase student understanding of complex processes when they are used in conjunction with activities that support comprehension.

*Improving Pedagogical Content Knowledge.* Professional development is considered an essential mechanism for deepening teachers' content knowledge and developing their teaching practices. Professional development programs aimed at the development of teachers' PCK and TPACK, should be closely aligned to teachers' professional practice. Professional development should engage teachers in aligning activities to specific mathematics content in their classrooms with the potential to develop teachers' TPACK and support integration of education technologies in their classrooms.

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